A Singapore Math Competition puzzle called **Cheryl's Birthday** raised a lot of interest and comments, reaching BBC, New York Times, Guardian and Washington Post science columns. The problem posed by Singapore and Asian School Math Olympiad Commission is as follows:

Albert and Bernard just become friends with Cheryl, and they want to know when her birthday is. Cheryl gives them a list of 10 possible dates.

May 15   May 16   May 19
June 17   June 18
July 14   July 16
August 14   August 15   August 17

Cheryl then tells Albert and Bernard separately the month and day of her birthday respectively.

Albert: I don't know when Cheryl's birthday is, but I know that Bernard does not know too.
Bernard: At first I don't know when Cheryl's birthday is, but I know now.
Albert: Then I also know when Cheryl's birthday is.

So when is Cheryl's birthday?

The Solution in Bipartite Graph Framework

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A convenient way to model the problem is with a bipartite graph consisting of A-nodes (left side), B-nodes (right side) and links connecting A-nodes with B-nodes. A-nodes represent a valid pool of months, B-nodes represent a valid pool of days and links represent valid combinations taken from Cheryl's list. Albert "owns" A-nodes but hides them from Bernard, Bernard "owns" B-nodes and hides them from Albert. So A-nodes are a guessing pool from Bernard's perspective and B-nodes are a guessing pool from Albert's perspective. As graph shows half of each participant's perspective, care is required to distinguish this.

Initial graph looks like this:

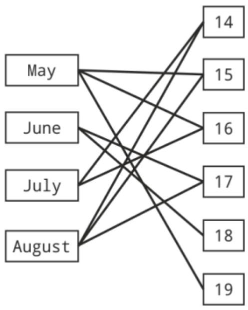

fig. 1

In next steps we will try to eliminate nodes and links based on statements given by Albert and Bernard and their knowledge at subsequent time points. Please note the special kind of nodes – *terminals* – with only single link connecting them to some part of the graph. We will watch the terminals and who owns them!

First statement (Albert's) has two parts "I don't know the answer" and "I know Bernard doesn't know the answer". First part is obvious and redundant because all of his nodes have multiple links. The second part is important. Albert is certain that 18 and 19 cannot be among B-nodes - these are terminal nodes and his opponent would know the answer at once given 18 or 19. After hearing Albert we can eliminate them from graph, deleting also their links which become invalid and unnecessary.

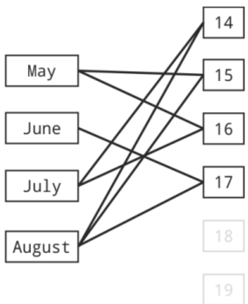

fig. 2

Hearing this statement, Bernard can draw his conclusions too. He can be certain that June is not among A-nodes, because otherwise his opponent would have a terminal node and since – the answer (Albert just said he had no answer). After Bernard's thoughts we can eliminate June and it's links, arriving at this graph:

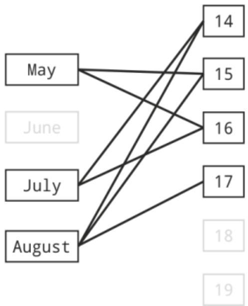

fig. 3

At this moment we are waiting for Bernard's statement, seeing only one terminal node among B-nodes owned by Bernard, which is 17. So if we hear "I know the answer" from Bernard now, it must be true that he has 17.

Second statement (Bernard's) confirms this: Bernard knows the answer, so his B-node must have been 17. Because 17 is only connected to August, **the correct answer is August 17**.

We have arrived at the answer without even referring to the third statement. This is not surprise: after the second statement anyone can solve the problem, so can Albert. Albert just acknowledged the fact known to everyone.

Discussion

The solution is only possible under a subtle assumption. The rule must hold that Cheryl cannot give an obvious day number (18 or 19) to Bernard, that is the information allowing one of participants to guess immediately. If this rule is missing, a contradiction appears (which is the case of the solution given by the Singapore and Asian School Math Olympiad Commission).

Let's go back to Albert's decision before his first statement and his thinking about non-obvious-date rule. If two obvious B-nodes are allowed, after hearing "I know Bernard doesn't know then answer" from Albert, we must assume that Albert did not have either May or June (and not only May as previously), because he felt necessary to exclude 18 and 19 completely from further processing. This leads to a smaller, but more difficult version of the third graph:

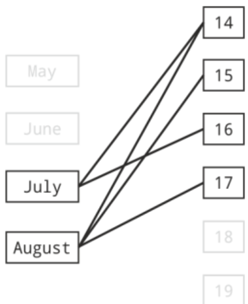

fig. 4

Under what circumstances could Bernard tell "I know the answer" now? It would be possible only if his day number was coincident with one of terminal nodes 15, 16 or 17. Thus we eliminate 14 from B-nodes and it's links and this is graph transformation after the second statement:

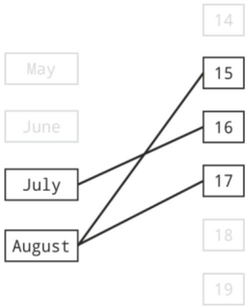

fig. 5

Now Albert comes across a problem. He can infer that correct answer is June 16 (terminals on both sides). But at the same time, he hears that Bernard didn't have the answer at the beginning, which is an evidence that Bernard was not given an obvious day. Albert cannot ignore this fact nor include it in this line of inference. However, having reached the dead end, Albert can still go back and re-evaluate his initial perspective (before first statement). He is now certain that *non-obvious-date-rule* is in place and transforms the graph in less restrictive way, already illustrated in fig. 1. Continuing this way Albert arrives at August 17 answer.

There are two key elements necessary to solve the problem: first is to notice the importance of non-obvious-date-rule, second is to free participants from the tyranny of linear order. Initial conditions do not impose any requirement for Albert or Bernard to push inference in one direction only. The linear order of events does not prohibit unlinear order of inference.

Description given above in form of two independent solutions, one correct and the other incorrect can be re-stated as a single two-branch process with one (incorrect) branch closing a cycle at initial fork and the other leading to successful solution. The choice is just a matter of stylistic preference.